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Super Yang-Mills theories on the two-dimensional lattice with exact supersymmetry

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Abstract: We construct super Yang-Mills theories with $\mathcal{N} = 2, 4$ supersymmetries on the two-dimensional square lattice keeping one or two supercharges exactly. Along the same line as the previous paper [1], the construction is based on topological field theory formulation. We present two kinds of modifications of the action which preserve the exact supersymmetry and resolve the problem of degenerate classical vacua encountered in the previous paper. Any supersymmetry breaking terms do not need to be introduced, and the formulations exactly realize some of supersymmetries at the lattice level. Our lattice actions flow to the desired continuum theories without any fine tuning of parameters.

Keywords: Topological Field Theories, Lattice Quantum Field Theory, Lattice Gauge Field Theories, Extended Supersymmetry.

Contents

1. Introduction

Lattice formulations of supersymmetric theories have a long history and still have continued to be vigorously investigated [2]–[9].[1] Recently, for super Yang-Mills (SYM) theories, an interesting approach motivated from the idea of deconstruction was presented [11, 12].[2] Also, various theories without gauge symmetry were discussed based on the connection to topological field theory [15].

In the previous paper [1], we constructed SYM theories with extended supersymmetry on the hypercubic lattices of various dimensions. The construction is based on topological field theory formulation [16, 17], and keeps one or two supercharges manifestly. The gauge fields are expressed as ordinary compact unitary variables on the lattice links. The lattice actions have a number of the classical vacua with the degeneracy growing as exponential of the number of plaquettes, which makes unclear their connection to the perturbative regime of the SYM theories. In order to resolve the degeneracy and to single out the vacuum corresponding to the target theory, we added to the action some supersymmetry breaking term which is tuned to vanish in the continuum limit.

In this paper, the same kind of lattice models are considered for $\mathcal{N}=2, 4$ supersymmetries in two-dimensions. We make suitable modifications to the action so that the vacuum degeneracy is completely resolved *with keeping the exact supersymmetry.* Two such modifications are presented here. One is a simple modification of the function $\Phi(x)$ in the lattice actions as $\Phi(x) \to \Phi(x) + \Delta\Phi(x)$, that works well for the gauge group SU(N). The other is to impose the so-called admissibility conditions on plaquette variables similar to those for gauge fields coupled to the Ginsparg-Wilson fermions [18]. It will be applicable to

[1]For a recent review, see [10].

[2]For related works, see [13, 14].

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more general cases not restricted to the SU(N) gauge group. Because both of modifications maintain the exact supersymmetry possessed by the original models in [1], we do not have to add any supersymmetry breaking terms. The supersymmetry is exactly realized at the lattice level in the SYM theories.

This paper is organized as follows. In section 2, we briefly review the lattice actions constructed in [1] for the $\mathcal{N} = 2, 4$ theories in two-dimensions. In section 3, we discuss on the modification $\Phi(x) \to \Phi(x) + \Delta\Phi(x)$ to resolve the problem of degenerate vacua. In section 4, we impose the admissibility conditions on plaquette variables, where the exact supersymmetry is preserved by doing a similar modification to the action as in ref. [19]. Section 5 is devoted to the summary and discussions.

Throughout this paper, we consider the gauge group $G = \mathrm{SU}(N)$ and the two-dimensional square lattice. Gauge fields are expressed as compact unitary variables

$$U_\mu(x) = e^{iaA_\mu(x)} \tag{1.1}$$

on the link $(x, x+\hat{\mu})$. 'a' stands for the lattice spacing, and $x \in \mathbb{Z}^2$ the lattice site. All other fields are put on the sites and expanded by a basis of $N \times N$ traceless hermitian matrices T^a $(a = 1, \cdots, N^2 - 1)$.

2. Brief review of lattice actions

We briefly review the lattice actions of two-dimensional $\mathcal{N} = 2, 4$ SYM theories discussed in [1].

2.1 $\mathcal{N} = 2$ case

Other than the gauge variables $U_\mu(x)$, the $\mathcal{N} = 2$ theory has complex scalars $\phi(x)$, $\bar{\phi}(x)$, and fermions are denoted as $\psi_\mu(x)$, $\chi(x)$, $\eta(x)$ [16]. They are transformed under the exact supersymmetry Q as

$$\begin{aligned}
QU_\mu(x) &= i\psi_\mu(x)U_\mu(x)\,,\\
Q\psi_\mu(x) &= i\psi_\mu(x)\psi_\mu(x) - i\left(\phi(x) - U_\mu(x)\phi(x+\hat{\mu})U_\mu(x)^\dagger\right),\\
Q\phi(x) &= 0\,,\\
Q\chi(x) &= H(x)\,, \qquad QH(x) = [\phi(x),\, \chi(x)]\,,\\
Q\bar{\phi}(x) &= \eta(x)\,, \qquad Q\eta(x) = [\phi(x),\, \bar{\phi}(x)]\,,
\end{aligned} \tag{2.1}$$

where $H(x)$ is an auxiliary fields. Q is nilpotent up to an infinitesimal gauge transformation with the parameter $\phi(x)$. In the expansion $H(x) = \sum_a H^a(x)T^a$, coefficients $H^a(x)$ are real. $\phi^a(x)$, $\bar{\phi}^a(x)$ are complex, and the fermionic variables $\psi^a_\mu(x)$, $\chi^a(x)$, $\eta^a(x)$ may be regarded as complexified Grassmann[3] to be compatible to the $\mathrm{U}(1)_R$ rotations (2.5). Notice that in the path integral $\phi^a(x)$ and $\bar{\phi}^a(x)$ can be treated as independent variables and that each of $H^a(x)$ is allowed to be shifted by a complex number. Thus, (2.1) is consistently closed in the path integral expression of the theory.

[3] A complexified Grassmann number takes the form: (complex number) $\times$ (real Grassmann number).

The lattice action is

$$S^{\rm LAT}_{\mathcal{N}=2} = Q\frac{1}{2g_0^2}\sum_x \mathrm{tr}\left[\frac{1}{4}\eta(x)\,[\phi(x),\,\bar{\phi}(x)] - i\chi(x)\Phi(x) + \chi(x)H(x) + \right.$$
$$\left. + i\sum_{\mu=1}^{2}\psi_\mu(x)\left(\bar{\phi}(x) - U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right)\right], \qquad (2.2)$$

where

$$\Phi(x) = -i\,[U_{12}(x) - U_{21}(x)]\,, \qquad (2.3)$$

$U_{\mu\nu}(x)$ are plaquette variables written as

$$U_{\mu\nu}(x) \equiv U_\mu(x)U_\nu(x+\hat{\mu})U_\mu(x+\hat{\nu})^\dagger U_\nu(x)^\dagger\,. \qquad (2.4)$$

The action (2.2) is clearly Q-invariant from its Q-exact form. Also, the invariance under the following global $\mathrm{U}(1)_R$ rotation holds:

$$\begin{aligned} U_\mu(x) &\to U_\mu(x)\,, & \psi_\mu(x) &\to e^{i\alpha}\psi_\mu(x)\,,\\ \phi(x) &\to e^{2i\alpha}\phi(x)\,, & &\\ \chi(x) &\to e^{-i\alpha}\chi(x)\,, & H(x) &\to H(x)\,,\\ \bar{\phi}(x) &\to e^{-2i\alpha}\bar{\phi}(x)\,, & \eta(x) &\to e^{-i\alpha}\eta(x)\,. \end{aligned} \qquad (2.5)$$

The $\mathrm{U}(1)_R$ charge of each variable is read off from (2.5), and Q increases the charge by one.

After acting Q in the r.h.s. of (2.2), the action is expressed as

$$S^{\rm LAT}_{\mathcal{N}=2} = \frac{1}{2g_0^2}\sum_x \mathrm{tr}\left[\frac{1}{4}[\phi(x),\,\bar{\phi}(x)]^2 + H(x)^2 - iH(x)\Phi(x) + \right.$$
$$+\sum_{\mu=1}^{2}\left(\phi(x) - U_\mu(x)\phi(x+\hat{\mu})U_\mu(x)^\dagger\right)\left(\bar{\phi}(x) - U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right) -$$
$$-\frac{1}{4}\eta(x)[\phi(x),\,\eta(x)] - \chi(x)[\phi(x),\,\chi(x)] -$$
$$-\sum_{\mu=1}^{2}\psi_\mu(x)\psi_\mu(x)\left(\bar{\phi}(x) + U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right) +$$
$$\left. + i\chi(x)Q\Phi(x) - i\sum_{\mu=1}^{2}\psi_\mu(x)\left(\eta(x) - U_\mu(x)\eta(x+\hat{\mu})U_\mu(x)^\dagger\right)\right]. \qquad (2.6)$$

Integration of $H(x)$ induces $\Phi(x)^2$ term, which yields the gauge kinetic term as the form

$$\frac{1}{2g_0^2}\sum_x \mathrm{tr}\left[-(U_{12}(x) - U_{21}(x))^2\right], \qquad (2.7)$$

which leads a number of the classical vacua

$$U_{12}(x) = \begin{pmatrix} \pm 1 & & \\ & \ddots & \\ & & \pm 1 \end{pmatrix} \qquad (2.8)$$

up to gauge transformations, where any combinations of ± 1 with '-1' appearing even times are allowed in the diagonal entries. Since an arbitrary configuration of (2.8) can be taken for each plaquette, it leads the degeneracy growing as exponential of the number of plaquettes. In order to investigate the dynamics of the model, we need to sum up contributions from all of the minima, and the ordinary weak field expansion around a single vacuum $U_{12}(x) = 1$ can not be justified.

2.2 $\mathcal{N} = 4$ case

In the $\mathcal{N} = 4$ theory, scalar fields $B(x)$, $C(x)$ appear as well as $\phi(x)$ and $\bar{\phi}(x)$. Auxiliary fields $\tilde{H}_\mu(x)$, $H(x)$ are introduced, and fermions are $\psi_\pm(x)$, $\chi_\pm(x)$, $\eta_\pm(x)$ [17].[4] The latticization keeps two supercharges $Q_\pm$, which transform the fields as

$$
\begin{aligned}
Q_+U_\mu(x) &= i\psi_{+\mu}(x)U_\mu(x)\,,\\
Q_-U_\mu(x) &= i\psi_{-\mu}(x)U_\mu(x)\,,\\
Q_+\psi_{+\mu}(x) &= i\psi_{+\mu}(x)\psi_{+\mu}(x) - i\left(\phi(x) - U_\mu(x)\phi(x+\hat{\mu})U_\mu(x)^\dagger\right),\\
Q_-\psi_{-\mu}(x) &= i\psi_{-\mu}(x)\psi_{-\mu}(x) + i\left(\bar{\phi}(x) - U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right),\\
Q_-\psi_{+\mu}(x) &= \frac{i}{2}\left\{\psi_{+\mu}(x),\,\psi_{-\mu}(x)\right\} - \frac{i}{2}\left(C(x) - U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right) - \tilde{H}_\mu(x)\,,\\
Q_+\psi_{-\mu}(x) &= \frac{i}{2}\left\{\psi_{+\mu}(x),\,\psi_{-\mu}(x)\right\} - \frac{i}{2}\left(C(x) - U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right) + \tilde{H}_\mu(x)\,,\\
Q_+\tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{-\mu}(x),\,\phi(x) + U_\mu(x)\phi(x+\hat{\mu})U_\mu(x)^\dagger\right] +\\
&\quad +\frac{1}{4}\left[\psi_{+\mu}(x),\,C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] +\\
&\quad +\frac{i}{2}\left(\eta_+(x) - U_\mu(x)\eta_+(x+\hat{\mu})U_\mu(x)^\dagger\right) +\\
&\quad +\frac{i}{2}\left[\psi_{+\mu}(x),\,\tilde{H}_\mu(x)\right] + \frac{1}{4}\left[\psi_{+\mu}(x)\psi_{+\mu}(x),\,\psi_{-\mu}(x)\right],\\
Q_-\tilde{H}_\mu(x) &= -\frac{1}{2}\left[\psi_{+\mu}(x),\,\bar{\phi}(x) + U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right] -\\
&\quad -\frac{1}{4}\left[\psi_{-\mu}(x),\,C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right] -\\
&\quad -\frac{i}{2}\left(\eta_-(x) - U_\mu(x)\eta_-(x+\hat{\mu})U_\mu(x)^\dagger\right) +\\
&\quad +\frac{i}{2}\left[\psi_{-\mu}(x),\,\tilde{H}_\mu(x)\right] - \frac{1}{4}\left[\psi_{-\mu}(x)\psi_{-\mu}(x),\,\psi_{+\mu}(x)\right], \qquad (2.9)\\
Q_+B(x) &= \chi_+(x)\,,\quad Q_+\chi_+(x) = [\phi(x),\,B(x)]\,,\quad Q_-\chi_+(x) = \frac{1}{2}[C(x),\,B(x)] - H(x)\,,\\
Q_-B(x) &= \chi_-(x)\,,\quad Q_-\chi_-(x) = -[\bar{\phi}(x),\,B(x)]\,,\quad Q_+\chi_-(x) = \frac{1}{2}[C(x),\,B(x)] + H(x)\,,\\
Q_+H(x) &= [\phi(x),\,\chi_-(x)] + \frac{1}{2}[B(x),\,\eta_+(x)] - \frac{1}{2}[C(x),\,\chi_+(x)]\,,
\end{aligned}
$$

[4] $B^a(x)$, $C^a(x)$, $\tilde{H}^a_\mu(x)$, $H^a(x)$ are real, and $\psi^a_\pm(x)$, $\chi^a_\pm(x)$, $\eta^a_\pm(x)$ may be regarded as complexified Grassmann.

$$\begin{aligned} Q_-H(x) &= [\bar{\phi}(x), \chi_+(x)] - \frac{1}{2}[B(x), \eta_-(x)] + \frac{1}{2}[C(x), \chi_-(x)] \,, \qquad (2.10) \\ Q_+C(x) &= \eta_+(x)\,, \quad Q_+\eta_+(x) = [\phi(x), C(x)]\,, \quad Q_-\eta_+(x) = -[\phi(x), \bar{\phi}(x)], \\ Q_-C(x) &= \eta_-(x)\,, \quad Q_-\eta_-(x) = -[\bar{\phi}(x), C(x)]\,, \quad Q_+\eta_-(x) = [\phi(x), \bar{\phi}(x)]\,, \\ Q_+\phi(x) &= 0\,, \quad Q_-\phi(x) = -\eta_+(x)\,, \quad Q_+\bar{\phi}(x) = \eta_-(x)\,, \quad Q_-\bar{\phi}(x) = 0\,. \qquad (2.11) \end{aligned}$$

The transformation leads the following nilpotency of $Q_\pm$ (up to gauge transformations):

$$\begin{aligned} Q_+^2 &= (\text{infinitesimal gauge transformation with the parameter } \phi)\,, \\ Q_-^2 &= (\text{infinitesimal gauge transformation with the parameter } -\bar{\phi})\,, \\ \{Q_+, Q_-\} &= (\text{infinitesimal gauge transformation with the parameter } C)\,. \qquad (2.12) \end{aligned}$$

We constructed the lattice action with exact $Q_\pm$ symmetry as

$$\begin{aligned} S^{\mathrm{LAT}}_{\mathcal{N}=4} = Q_+Q_-\frac{1}{2g_0^2}\sum_x \mathrm{tr}\Bigg[&- iB(x)\Phi(x) - \sum_{\mu=1}^{2}\psi_{+\mu}(x)\psi_{-\mu}(x) - \\ &- \chi_+(x)\chi_-(x) - \frac{1}{4}\eta_+(x)\eta_-(x)\Bigg]\,, \qquad (2.13) \end{aligned}$$

where $\Phi(x)$ is given by (2.3). The action is invariant under the $\mathrm{SU}(2)_R$ transformation, whose generators are expressed as

$$\begin{aligned} J_{++} &= \sum_{x,\,a}\Bigg[\sum_\mu \psi^a_{+\mu}(x)\frac{\partial}{\partial\psi^a_{-\mu}(x)} + \chi^a_+(x)\frac{\partial}{\partial\chi^a_-(x)} - \eta^a_+(x)\frac{\partial}{\partial\eta^a_-(x)} + \\ &\quad + 2\phi^a(x)\frac{\partial}{\partial C^a(x)} - C^a(x)\frac{\partial}{\partial\bar{\phi}^a(x)}\Bigg]\,, \\ J_{--} &= \sum_{x,\,a}\Bigg[\sum_\mu \psi^a_{-\mu}(x)\frac{\partial}{\partial\psi^a_{+\mu}(x)} + \chi^a_-(x)\frac{\partial}{\partial\chi^a_+(x)} - \eta^a_-(x)\frac{\partial}{\partial\eta^a_+(x)} - \\ &\quad - 2\bar{\phi}^a(x)\frac{\partial}{\partial C^a(x)} + C^a(x)\frac{\partial}{\partial\phi^a(x)}\Bigg]\,, \\ J_0 &= \sum_{x,\,a}\Bigg[\sum_\mu \psi^a_{+\mu}(x)\frac{\partial}{\partial\psi^a_{+\mu}(x)} - \sum_\mu \psi^a_{-\mu}(x)\frac{\partial}{\partial\psi^a_{-\mu}(x)} + \chi^a_+(x)\frac{\partial}{\partial\chi^a_+(x)} - \\ &\quad - \chi^a_-(x)\frac{\partial}{\partial\chi^a_-(x)} + \eta^a_+(x)\frac{\partial}{\partial\eta^a_+(x)} - \eta^a_-(x)\frac{\partial}{\partial\eta^a_-(x)} + \\ &\quad + 2\phi^a(x)\frac{\partial}{\partial\phi^a(x)} - 2\bar{\phi}^a(x)\frac{\partial}{\partial\bar{\phi}^a(x)}\Bigg]\,, \qquad (2.14) \end{aligned}$$

with the script a being the index of a basis of the gauge group generators. They form the SU(2) algebra:

$$[J_0, J_{++}] = 2J_{++}\,, \qquad [J_0, J_{--}] = -2J_{--}\,, \qquad [J_{++}, J_{--}] = J_0\,. \qquad (2.15)$$

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Under the $\mathrm{SU}(2)_R$, each of $(\psi^a_{+\mu}, \psi^a_{-\mu})$, (χ^a_+, χ^a_-), $(\eta^a_+, -\eta^a_-)$ and (Q_+, Q_-) transforms as a doublet, and $(\phi^a, C^a, -\bar{\phi}^a)$ as a triplet. We can easily see the $\mathrm{SU}(2)_R$ invariance of the action (2.13), because Q_+Q_-, $\mathrm{tr}(\psi_{+\mu}(x)\psi_{-\mu}(x))$, $\mathrm{tr}(\chi_+(x)\chi_-(x))$ and $\mathrm{tr}(\eta_+(x)\eta_-(x))$ are $\mathrm{SU}(2)_R$ singlets. Also, the action has the symmetry of exchanging the two supercharges $Q_+ \leftrightarrow Q_-$ with

$$\begin{aligned} &\phi \to -\bar{\phi}\,, &&\bar{\phi} \to -\phi\,, &&B \to -B\,, \\ &\chi_+ \to -\chi_-\,, &&\chi_- \to -\chi_+\,, &&\tilde{H}_\mu \to -\tilde{H}_\mu\,, \\ &\psi_{\pm\mu} \to \psi_{\mp\mu}\,, &&\eta_\pm \to \eta_\mp\,. && \end{aligned} \tag{2.16}$$

After acting $Q_\pm$, the action is more explicitly written as

$$\begin{aligned} S^{2d\,\mathrm{LAT}}_{\mathcal{N}=4} = \frac{1}{2g_0^2}\sum_x \mathrm{tr}\Bigg[&-i\left(\frac{1}{2}[C(x),\,B(x)] + H(x)\right)\Phi(x) + H(x)^2 + i\chi_-(x)Q_+\Phi(x) - \\ &- i\chi_+(x)Q_-\Phi(x) - iB(x)Q_+Q_-\Phi(x) - [\phi(x),\,B(x)][\bar{\phi}(x),\,B(x)] - \\ &- \frac{1}{4}[C(x),\,B(x)]^2 + \chi_+(x)[\bar{\phi}(x),\,\chi_+(x)] - \chi_-(x)[\phi(x),\,\chi_-(x)] + \\ &+ \chi_-(x)[C(x),\,\chi_+(x)] - \chi_-(x)[B(x),\,\eta_+(x)] - \\ &- \chi_+(x)[B(x),\,\eta_-(x)] - \frac{1}{4}[\phi(x),\,\bar{\phi}(x)]^2 - \\ &- \frac{1}{4}[\phi(x),\,C(x)][\bar{\phi}(x),\,C(x)] - \frac{1}{4}\eta_-(x)[\phi(x),\,\eta_-(x)] + \\ &+ \frac{1}{4}\eta_+(x)[\bar{\phi}(x),\,\eta_+(x)] - \frac{1}{4}\eta_+(x)[C(x),\,\eta_-(x)]\Bigg] + \\ +\frac{1}{2g_0^2}\sum_x\sum_{\mu=1}^{2}\mathrm{tr}\Bigg[&\tilde{H}_\mu(x)^2 - \frac{1}{2}\psi_{+\mu}(x)\psi_{+\mu}(x)\psi_{-\mu}(x)\psi_{-\mu}(x) + \\ &+ \left(\phi(x) - U_\mu(x)\phi(x+\hat{\mu})U_\mu(x)^\dagger\right)\times \\ &\times\left(\bar{\phi}(x) - U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right) + \\ &+ \frac{1}{4}\left(C(x) - U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right)^2 - \\ &- \psi_{+\mu}(x)\psi_{+\mu}(x)\left(\bar{\phi}(x) + U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right) + \\ &+ \psi_{-\mu}(x)\psi_{-\mu}(x)\left(\phi(x) + U_\mu(x)\phi(x+\hat{\mu})U_\mu(x)^\dagger\right) \\ &- i\psi_{+\mu}(x)\left(\eta_-(x) - U_\mu(x)\eta_-(x+\hat{\mu})U_\mu(x)^\dagger\right) - \\ &- i\psi_{-\mu}(x)\left(\eta_+(x) - U_\mu(x)\eta_+(x+\hat{\mu})U_\mu(x)^\dagger\right) - \\ &- \frac{1}{2}\{\psi_{+\mu}(x),\,\psi_{-\mu}(x)\}\left(C(x) + U_\mu(x)C(x+\hat{\mu})U_\mu(x)^\dagger\right)\Bigg]. \end{aligned} \tag{2.17}$$

Similarly to the $\mathcal{N}=2$ case, the gauge kinetic term, induced after integrating $H(x)$ out, suffers from the problem of degenerate classical vacua.

In the previous paper [1], for both cases of $\mathcal{N}=2$, 4 theories in two-dimensional $M \times M$ periodic lattice, we added the term

$$\Delta S = \frac{1}{2g_0^2}\,\rho \sum_x \mathrm{tr}\,(2 - U_{12}(x) - U_{21}(x)) \tag{2.18}$$

with $\rho = \frac{1}{M^s}$ $(0 < s < 2)$ to resolve the vacuum degeneracy. This term breaks the exact supersymmetries Q and $Q_\pm$ respectively, but it is tuned to vanish in the continuum limit. In what follows, we will consider to resolve the degeneracy keeping the exact supersymmetries without introducing ΔS.

3. Modification of $\Phi(x)$

In order to single out the vacuum $U_{12}(x) = 1$ from (2.8), we add a term $\Delta\Phi(x)$ to $\Phi(x)$:

$$\Phi(x) \to \Phi(x) + \Delta\Phi(x)\,, \qquad \Delta\Phi(x) \equiv -r(2 - U_{12}(x) - U_{21}(x))\,. \tag{3.1}$$

There are many choices for the parameter r. First we shall take as

$$r = \cot\frac{\pi}{2N}\,. \tag{3.2}$$

For other choices, we will discuss later. Because the modification does not affect the Q-exact structure of the actions, the exact supersymmetry Q is preserved for $\mathcal{N}=2$ case. Also the $\mathrm{U}(1)_R$ symmetry is kept intact, because the link variables do not transform under the $\mathrm{U}(1)_R$ transformation. For $\mathcal{N}=4$ case, from the same reasons, the modification keeps the $Q_\pm$ supersymmetry as well as the $\mathrm{SU}(2)_R$ and $Q_+ \leftrightarrow Q_-$ symmetries (2.14), (2.16).

The classical vacua are determined by

$$\Phi(x) + \Delta\Phi(x) = 0\,. \tag{3.3}$$

$U_{12}(x)$ can be expressed as the diagonalized form

$$U_{12}(x) = \Omega(x) \begin{pmatrix} e^{i\theta_1(x)} & & \\ & \ddots & \\ & & e^{i\theta_N(x)} \end{pmatrix} \Omega(x)^\dagger \tag{3.4}$$

with $\Omega(x) \in \mathrm{SU}(N)$, $\theta_N(x) \equiv -\theta_1(x) - \cdots - \theta_{N-1}(x)$, and $-\pi < \theta_i(x) \leq +\pi$ $(i = 1, \cdots, N-1)$. Under this parameterization, the equation for the minima (3.3) reads

$$\begin{aligned} \sin\theta_1(x) &= 2r\sin^2\left(\frac{\theta_1(x)}{2}\right), \\ &\vdots \\ \sin\theta_{N-1}(x) &= 2r\sin^2\left(\frac{\theta_{N-1}(x)}{2}\right), \\ -\sin(\theta_1(x) + \cdots + \theta_{N-1}(x)) &= 2r\sin^2\left(\frac{\theta_1(x) + \cdots + \theta_{N-1}(x)}{2}\right). \end{aligned} \tag{3.5}$$

With the choice (3.2), the first $N-1$ equations give the solutions $\theta_i(x) = 0$ or $\frac{\pi}{N}$ ($i = 1, \cdots, N-1$). Among them, the last equation allows the only one combination

$$(\theta_1(x), \ldots, \theta_{N-1}(x)) = (0, \ldots, 0)\,, \tag{3.6}$$

which is nothing but $U_{12}(x) = 1$. (Any other combinations of the solutions are not compatible to the last equation, because the l.h.s. is evaluated to be negative while the r.h.s. positive.)

Let us discuss about other choices of r. Parameterizing as $r = \cot\varphi$,

$$\Phi(x) + \Delta\Phi(x) = \frac{-1}{\sin\varphi}\left[e^{-i\varphi}(1 - U_{12}(x)) + e^{i\varphi}(1 - U_{21}(x))\right], \tag{3.7}$$

where $\Phi(x) + \Delta\Phi(x) = 0$ means that $e^{-i\varphi}(1 - U_{12}(x))$ is anti-hermitian. Configurations of $U_{12}(x)$ giving the minima are expressed as

$$U_{12}(x) = 1 - ie^{i\varphi}T(x) \tag{3.8}$$

with $T(x)$ being hermitian. The eigenvalues of $T(x)$ are denoted as $t_i(x)$ ($i = 1, \cdots, N$). The unitary condition $U_{12}(x)U_{12}(x)^\dagger = 1$ determines $t_i(x)$ as

$$t_i(x) = 0 \qquad \text{or} \qquad -2\sin\varphi \qquad \text{for } i = 1, \ldots, N\,. \tag{3.9}$$

The unitarity alone does not uniquely fix $t_i(x)$. However, the unimodular condition

$$\det U_{12}(x) = 1 \tag{3.10}$$

gives further constraints. For the case that ℓ eigenvalues of $t_i(x)$ are $-2\sin\varphi$ and the remaining $N - \ell$ are 0, (3.10) leads

$$e^{i2\ell\varphi} = 1\,. \tag{3.11}$$

Thus, taking φ such that

$$e^{i2\ell\varphi} \neq 1 \qquad \text{for } {}^\forall\ell = 1, \ldots, N\,, \tag{3.12}$$

the equation $\Phi(x) + \Delta\Phi(x) = 0$ has the unique solution $t_1(x) = \cdots = t_N(x) = 0$, equivalent to $U_{12}(x) = 1$. Of course, the choice (3.2) satisfies (3.12).

No fermion doublers and renormalization. For $\mathcal{N} = 2$ case, under the modification (3.1) with $r = \cot\varphi$ satisfying (3.12), our lattice action (2.6) has the unique minimum $U_{12}(x) = 1$, which justifies expanding the exponential of the link variables (1.1). Note that the modification $\Delta\Phi(x)$ does not affect the classical continuum limit:

$$a \to 0 \qquad \text{with } g_2^2 \equiv \frac{g_0^2}{a^2} \qquad \text{fixed}\,. \tag{3.13}$$

Compared with $\Phi(x)$ giving $O(a^2)$ contributions, $\Delta\Phi(x)$ is of the negligible order $O(a^4)$. Furthermore, it turns out that the shift of the fermionic part of the action

$$\mathrm{tr}[i\chi(x)Q\Delta\Phi(x)] = \mathrm{tr}\left[ir\chi(x)\left(QU_{12}(x) + QU_{21}(x)\right)\right] \tag{3.14}$$

leads gauge-fermion interaction terms of the irrelevant order $O(a^5)$, but not fermion kinetic terms. (Notice that fermionic variables are rescaled as (fermions) $\to a^{3/2}$(fermions) when taking the continuum limit.) Thus, contribution to the fermion kinetic terms is not influenced by the modification, and no fermion doublers appear from the same argument as in the previous paper [1]. Indeed, the fermion kinetic terms are expressed as

$$S_f^{(2)} = \frac{a^4}{2g_0^2}\sum_{x,\mu}\mathrm{tr}\left[-\frac{1}{2}\Psi(x)^T\gamma_\mu(\Delta_\mu+\Delta_\mu^*)\Psi(x) - a\frac{1}{2}\Psi(x)^T P_\mu\Delta_\mu\Delta_\mu^*\Psi(x)\right], \tag{3.15}$$

where fermions were combined as $\Psi^T = \left(\psi_1, \psi_2, \chi, \frac{1}{2}\eta\right)$. The γ-matrices and P_μ are given by

$$\gamma_1 = -i\begin{pmatrix} 0 & \sigma_1 \\ \sigma_1 & 0 \end{pmatrix}, \qquad \gamma_2 = i\begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix}, \qquad P_1 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \qquad P_2 = -i\begin{pmatrix} 0 & \mathbf{1}_2 \\ -\mathbf{1}_2 & 0 \end{pmatrix} \tag{3.16}$$

with σ_i $(i = 1, 2, 3)$ being Pauli matrices. Note that they all anticommute each other:

$$\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}, \qquad \{P_\mu, P_\nu\} = 2\delta_{\mu\nu}, \qquad \{\gamma_\mu, P_\nu\} = 0. \tag{3.17}$$

Δ_μ and Δ_μ^* represent forward and backward difference operators respectively:

$$\Delta_\mu f(x) \equiv \frac{1}{a}\left(f(x+\hat{\mu}) - f(x)\right), \quad \Delta_\mu^* f(x) \equiv \frac{1}{a}\left(f(x) - f(x-\hat{\mu})\right). \tag{3.18}$$

The kernel of the kinetic terms (3.15) is written in the momentum space $-\pi/a \le q_\mu < \pi/a$ as

$$D = \sum_{\mu=1}^{2}\left[-i\gamma_\mu\frac{1}{a}\sin(q_\mu a) + 2P_\mu\frac{1}{a}\sin^2\left(\frac{q_\mu a}{2}\right)\right]. \tag{3.19}$$

It is easy to see that the kernel D vanishes only at the origin $q_1 = q_2 = 0$, because using (3.17) we get

$$D^2 = \frac{1}{a^2}\sum_{\mu=1}^{2}\left[\sin^2(q_\mu a) + 4\sin^4\left(\frac{q_\mu a}{2}\right)\right]. \tag{3.20}$$

Thus, the fermion kinetic terms contain no fermion doublers.

For the renormalization, we can repeat the argument in [1] without introducing the supersymmetry breaking term ΔS. Gauge symmetry and the $\mathrm{U}(1)_R$ invariance allow the operator $\mathrm{tr}\,\phi\bar{\phi}$, while it is forbidden by the supersymmetry Q. Hence, radiative corrections do not generate relevant or marginal operators except the identity. Our modified lattice action is shown to flow to the desired continuum theory without any fine tuning.

For $\mathcal{N} = 4$ case, the arguments go completely parallel. The fermion kinetic terms are represented in the same form as (3.15) with $\Psi^T = (\psi_{+1}, \psi_{+2}, \chi_+, \frac{1}{2}\eta_+, \psi_{-1}, \psi_{-2}, \chi_-, \frac{1}{2}\eta_-)$

and

$$\gamma_1 = -i\begin{pmatrix} & & & \sigma_1 \\ & & -i\sigma_2 & \\ & i\sigma_2 & & \\ \sigma_1 & & & \end{pmatrix}, \qquad \gamma_2 = -i\begin{pmatrix} & & & -\sigma_3 \\ & & \mathbf{1}_2 & \\ & \mathbf{1}_2 & & \\ -\sigma_3 & & & \end{pmatrix},$$
$$P_1 = -i\begin{pmatrix} & & & i\sigma_2 \\ & & -\sigma_1 & \\ & \sigma_1 & & \\ i\sigma_2 & & & \end{pmatrix}, \qquad P_2 = -i\begin{pmatrix} & & & \mathbf{1}_2 \\ & & \sigma_3 & \\ & -\sigma_3 & & \\ -\mathbf{1}_2 & & & \end{pmatrix}. \tag{3.21}$$

The matrices satisfy the anticommuting relation (3.17) again, which leads (3.20) indicating nonexistence of fermion doublers.

Gauge invariance and the $\mathrm{SU}(2)_R$ symmetry allow the operators $\mathrm{tr}(4\phi\bar{\phi}+C^2)$ and $\mathrm{tr}\, B^2$, but they are not admissible from the supersymmetry $Q_\pm$. Thus, no relevant or marginal operators appear through the loop effect except the identity operator. The continuum $\mathcal{N}=4$ theory is obtained with no tuning of parameters.

In this modification, the unimodular condition of $\mathrm{SU}(N)$ has an important role to single out the vacuum $U_{12}(x) = 1$. This kind of modification would not be useful for other gauge groups such as $\mathrm{U}(N)$. In the next section, we consider another modification, which will be applicable to more general cases, by imposing the admissibility conditions on plaquette variables. The $G = \mathrm{U}(N)$ case is not highly motivated from continuum SYM theories, because of the degrees of freedom corresponding to the $\mathrm{U}(1)$ factor decoupled. Almost the interesting dynamics is involved in the remaining $\mathrm{SU}(N)$ part. It is however certainly meaningful to present another kind of modification in the lattice models.

4. Admissibility conditions

As another modification to single out the vacuum $U_{12}(x) = 1$ from (2.8), we impose the so-called admissibility condition

$$||1 - U_{12}(x)|| < \epsilon \tag{4.1}$$

on each plaquette variable. For definiteness, we use the following definition of the norm of a matrix A:

$$||A|| \equiv \left[\mathrm{tr}\left(AA^\dagger\right)\right]^{1/2}, \tag{4.2}$$

and then ϵ is a positive number chosen as[5] $0 < \epsilon < 2\sqrt{2}$. The same kind of condition was introduced for gauge fields coupled to the Ginsparg-Wilson fermions [18]. Notice that (4.1) is a gauge invariant statement and that $||1 - U_{12}(x)||^2$ is proportional to the standard Wilson action:

$$||1 - U_{12}(x)||^2 = \mathrm{tr}\left[2 - U_{12}(x) - U_{21}(x)\right]. \tag{4.3}$$

[5] If we apply for $G = \mathrm{U}(N)$ case, ϵ should be chosen as $0 < \epsilon < 2$, because any combination of ± 1 in (2.8) is allowed with no restriction to the number of '-1'.

We modify the action of the $\mathcal{N}=2$ theory as follows: when $||1-U_{12}(x)||<\epsilon$,

$$\hat{S}^{\rm LAT}_{\mathcal{N}=2} = Q\frac{1}{2g_0^2}\sum_x \mathrm{tr}\left[\frac{1}{4}\eta(x)\,[\phi(x),\,\bar{\phi}(x)] - i\chi(x)\hat{\Phi}(x) + \chi(x)H(x) + \right.$$
$$\left. + i\sum_{\mu=1}^{2}\psi_\mu(x)\left(\bar{\phi}(x) - U_\mu(x)\bar{\phi}(x+\hat{\mu})U_\mu(x)^\dagger\right)\right], \tag{4.4}$$

and when $||1-U_{12}(x)||\geq\epsilon$,

$$\hat{S}^{\rm LAT}_{\mathcal{N}=2} = +\infty\,. \tag{4.5}$$

Here

$$\hat{\Phi}(x) = \frac{\Phi(x)}{1-\frac{1}{\epsilon^2}||1-U_{12}(x)||^2}\,. \tag{4.6}$$

The form of the action is somewhat similar to that of U(1) gauge theory constructed by Lüscher [19]. Note that the Boltzmann weight $\exp\left[-\hat{S}^{\rm LAT}_{\mathcal{N}=2}\right]$ is a product of local factors, which guarantees the locality of the theory. Also, it is easily seen that the Boltzmann weight is smooth and differentiable with respect to lattice variables for the region except the boundary

$$||1-U_{12}(x)|| = \epsilon\,. \tag{4.7}$$

Let us look closer at the smoothness on the boundary. For the partition function

$$Z = \int [\mathrm{d}\,(\mathrm{fields})]\exp\left[-\hat{S}^{\rm LAT}_{\mathcal{N}=2}\right], \tag{4.8}$$

after the integration over $H(x)$ and fermions, the relevant parts of the Boltzmann weight are evaluated as

$$\sum_{\{c(x)\}} d(\{c(x)\})\prod_x \mathcal{B}(c(x))\,, \tag{4.9}$$

$$\mathcal{B}(c(x)) \equiv \left(\frac{1}{1-\frac{1}{\epsilon^2}||1-U_{12}(x)||^2}\right)^{c(x)} \exp\left[-\frac{1}{8g_0^2}\frac{\mathrm{tr}(2-U_{12}(x)^2-U_{21}(x)^2)}{\left(1-\frac{1}{\epsilon^2}||1-U_{12}(x)||^2\right)^2}\right] \tag{4.10}$$

near the boundary. Here, $c(x)$ takes N^2-1 or $2(N^2-1)$ for each x, and $d(\{c(x)\})$ are irrelevant factors. It is smooth and differentiable on the boundary, which is essentially same as the fact that the function

$$f(t) = \begin{cases} \frac{1}{t^n}e^{-c/t^2} & \text{for } t>0 \\ 0 & \text{for } t\leq 0 \end{cases} \tag{4.11}$$

with c positive constant is smooth and differentiable at $t=0$ for $n=0,1,2,\ldots$. Similarly, for the unnormalized correlation function among the finite number of operators $\mathcal{O}_1,\cdots,\mathcal{O}_k$:

$$\int [\mathrm{d}\,(\mathrm{fields})]\,\mathcal{O}_1\cdots\mathcal{O}_k\,\exp\left[-\hat{S}^{\rm LAT}_{\mathcal{N}=2}\right], \tag{4.12}$$

the Boltzmann weight is smooth and differentiable on the boundary (4.7), as long as the operators contain the finite number of $H(x)$ for each x. (Compared to the case of the

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partition function, all the essential difference is that some of the powers $c(x)$ in (4.10) are shifted by finite amount, which does not affect the smoothness.) It leads the Q invariance of the Boltzmann weight as the following form:

$$\int [\mathrm{d}\,(\text{fields})]\, \mathcal{O}_1 \cdots \mathcal{O}_k Q \left(\exp\left[-\hat{S}^{\mathrm{LAT}}_{\mathcal{N}=2} \right] \right) = 0\,. \tag{4.13}$$

If we simply imposed (4.1) just by putting the step functions

$$\prod_x \theta\left(\epsilon^2 - ||1 - U_{12}(x)||^2\right) \tag{4.14}$$

in front of the Boltzmann weight $\exp\left[-S^{\mathrm{LAT}}_{\mathcal{N}=2}\right]$ with the original action (2.2), the supersymmetry Q would be broken due to the contribution from the boundary (4.7). The modification of the action (4.4), (4.5) however makes the breaking effect completely suppressed, and maintains the supersymmetry.

We may expand the exponential of the link variable (1.1), and the action leads to the $\mathcal{N} = 2$ SYM theory in the classical continuum limit (3.13). Note that ϵ is independent of the lattice spacing a. Also, the modification to the fermionic part of the action reads

$$\begin{aligned} \mathrm{tr}\left[i\chi(x)\, Q\hat{\Phi}(x) \right] &= \frac{1}{1 - \frac{1}{\epsilon^2}||1 - U_{12}(x)||^2}\, \mathrm{tr}\left[i\chi(x)\, Q\Phi(x) \right] - \\ &\quad - \frac{\mathrm{tr}\left[i\chi(x)\Phi(x) \right]}{\left(1 - \frac{1}{\epsilon^2}||1 - U_{12}(x)||^2\right)^2} \frac{1}{\epsilon^2}\, \mathrm{tr}\left[QU_{12}(x) + QU_{21}(x) \right], \end{aligned} \tag{4.15}$$

where the second term contributes to gauge-fermion interaction terms of the irrelevant order $O(a^8)$ but not to fermion kinetic terms. Thus, the modification does not affect the fermion kinetic terms, and the absence of fermion doubling is shown again as in the previous section. Since the $\mathrm{U}(1)_R$ symmetry is kept intact under the modification, we can do the same renormalization argument as in the previous section to conclude that the desired continuum theory is obtained without any fine tuning.

For the case of $\mathcal{N} = 4$ theory, similar modification is possible: when $||1 - U_{12}(x)|| < \epsilon$,

$$\begin{aligned} \hat{S}^{\mathrm{LAT}}_{\mathcal{N}=2} = Q_+ Q_- \frac{1}{2g_0^2} \sum_x \mathrm{tr} \Bigg[&- iB(x)\hat{\Phi}(x) - \sum_{\mu=1}^{2} \psi_{+\mu}(x)\psi_{-\mu}(x) - \\ &- \chi_+(x)\chi_-(x) - \frac{1}{4}\eta_+(x)\eta_-(x) \Bigg], \end{aligned} \tag{4.16}$$

and when $||1 - U_{12}(x)|| \geq \epsilon$,

$$\hat{S}^{\mathrm{LAT}}_{\mathcal{N}=4} = +\infty\,. \tag{4.17}$$

$\hat{\Phi}(x)$ is same as (4.6). Similarly to the $\mathcal{N} = 2$ case, the locality of the theory is satisfied, and the Boltzmann weight $\exp\left[-\hat{S}^{\mathrm{LAT}}_{\mathcal{N}=4}\right]$ is smooth and differentiable. It leads the $Q_\pm$ invariance of the Boltzmann weight in the same sense as (4.13). Also, the $\mathrm{SU}(2)_R$ and $Q_+ \leftrightarrow Q_-$ symmetries (2.14), (2.16) are not influenced by the modification. Arguments about the classical continuum limit, the absence of fermion doubling and the renormalization go without introducing the supersymmetry breaking term ΔS. The modified lattice action gives the desired continuum theory without any tuning of parameters.

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5. Summary and discussions

In this paper, two-dimensional SU(N) super Yang-Mills theories with $\mathcal{N} = 2, 4$ supersymmetries have been constructed on the square lattice, keeping one or two supercharges exactly. We have resolved a problem of the degenerate classical vacua, which was encountered in the previous paper [1], *with keeping the exact supersymmetry.* Thus, any supersymmetry breaking terms do not need to be introduced, and the formulations exactly realize the supersymmetries Q or $Q_\pm$ at the lattice level. Our lattice models define the continuum SYM theories without any fine tuning.

We have presented two different kinds of modifications of the actions to resolve the problem. One is a simple modification to add the term $\Delta\Phi(x)$ to the function $\Phi(x)$, where the unimodular condition of the gauge group SU(N) is crucial to single out the vacuum $U_{12}(x) = 1$. The other is to impose the admissibility condition on each plaquette variable $U_{12}(x)$ with changing the action somewhat analogous to ref. [19]. It will be also applicable to other gauge groups with an appropriate choice of ϵ. It would be worth while to pursue the direction of the second modification to discuss topological structures of the space of the admissible lattice gauge fields as in [19].

It is interesting to consider appropriate modifications to four-dimensional models with

$$\Phi_{\mathtt{A}}(x) = -i\left[U_{\mathtt{A}4}(x) - U_{4\mathtt{A}}(x) + \frac{1}{2}\sum_{\mathtt{B},\mathtt{C}=1}^{3}\varepsilon_{\mathtt{ABC}}\left(U_{\mathtt{BC}}(x) - U_{\mathtt{CB}}(x)\right)\right], \tag{5.1}$$

which correspond to $\mathcal{N} = 2, 4$ theories [20]. Even if suitable modifications are found and exact supersymmetries are realized at the lattice level, generically four-dimensional models would need some fine tuning of parameters to define the desired continuum limit. For the $\mathcal{N} = 4$ case, however the situation might seem to be subtle, because it is believed to receive no quantum corrections in any order of the perturbation theory (for example, see [21]). It would be intriguing to compute radiative corrections in the lattice perturbation theory for the modified $\mathcal{N} = 4$ model with the exact $Q_\pm$ supersymmetry.

Since the same kind of degeneracy problem exists in models proposed in ref. [4], the methods discussed here might be useful to resolve the difficulty there.

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